ON CORE-TO-PARTICLE INTERACTION FROM CLASSICAL POINT OF VIEW

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ABSTRACT. The treatment of the extra-core particle in the semi-rigid model of nucleus, in a core plus one particle system, in a sense justifies the form of the interaction hamiltonian used, in such cases, by A. Bohr in his collective model of nucleus.

INTRODUCTION

In the collective model of nucleus (A. Bohr, 1952) one takes the interaction between a particle and the core, in a core plus one particle system, to be confined only on the surface, the interaction being represented by

$$ {}_1H_{int} = -k\delta(r-R)\sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}\, Y^{\lambda}{}_{\mu}\,(\theta,\phi) \qquad \ldots \quad (1)$$

and as such the extra-core particle experiences no force when it goes within the nucleus. In this short note we have tried to find the order of magnitude of the force experienced by the extra-core particle when it goes within the liquid core, and to derive it we have followed a semi-rigid model for the whole system i.e. the rigid body motion of the extra-core particle and irrotational motion of the core. Classically the problem reduces to one of a small rigid sphere within a large volume of pulsating liquid of almost sperical shape and to seek an irrotational motion of the liquid consistent with the boundary conditions. We have found that the force experienced by the small sphere is derivable from a potential function and has exactly the form of an anisotropic harmonic-oscillator. We have also calculated the angular momentum and the magnetic moment of the system with the presence of the sphere within the nuclear fluid, and obtained a few correction terms in both cases over and above what A. Bohr (1952) has obtained for the same.

1. DERIVATION OF THE INTERACTION

Let the equation of the surface defining the boundary of the nucleus be given by

$$F \equiv R - R_0\left\{1+\frac{\alpha_{00}}{\sqrt{4\pi}} + \sum_{\mu=-2}^{2}\alpha_{2\mu}\, Y_{\mu}{}^{2}\,(\theta,\phi)\right\} = 0 \qquad \ldots \quad (2)$$

The condition at the boundary of the surface is

$$\frac{dF}{dt} = 0$$

or

$$\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial r} + \frac{v}{r}\frac{\partial F}{\partial \theta} + \frac{w}{r\sin\theta}\frac{\partial F}{\partial \phi} = 0 \qquad \dots \quad (3)$$

where u, v, w are the components of velocity in polar co-ordinates and the left hand side of (3) is evaluated at the surface given by (2).

Let us assume that the velocity potential at any point is given by

$$\phi = \sum_{\mu=-2}^{2} \beta_{2\mu}\, r^2 Y^2{}_{\mu}(\theta, \phi) \qquad \dots \quad (4)$$

Then substituting for u, v, w in (3) by those calculated from (4), it is easily seen that the first order solution of the resulting equation is

$$\beta_{2\mu} = -\tfrac{1}{2}\,\dot{\alpha}_{2\mu}$$

or

$$\phi = -\tfrac{1}{2}\sum_{\mu=-2}^{2} \dot{\alpha}_{2\mu}\, r^2\, Y^2{}_{\mu}(\theta, \phi) \qquad \dots \quad (5)$$

Now if a small sphere of radius δ is introduced within the fluid then the velocity potential given by (5) will be changed.

To facilitate the calculation of drag on this small sphere we change the origin to the centre of the sphere, without changing the direction of axes. Let (a, b, c) or (r_p, θ_p, ϕ_p) be the co-ordinates of the sphere referred to old system, and let (r', θ', ϕ') be the co-ordinate of any point in the new system which was (r, θ, ϕ) in the old system. Then equation (5) reduces to

$$\phi = -\tfrac{1}{2}\, r'^2 \sum_{\mu=-2}^{2} \dot{\alpha}_{2\mu} Y^2{}_{\mu}(\theta', \phi') - \tfrac{1}{2}\, r^2{}_p \sum_{\mu=-2}^{2} \dot{\alpha}_{2\mu}\, Y^2{}_{\mu}(\theta_p, \phi_p)$$

$$- r' \sum^{1} A_{\mu'}\, Y^1{}_{\mu'}(\theta', \phi') \qquad (6)$$

where

$$A_{\mu'} = r_p \sum_{m=-1} C^{\mu'}{}_m \dot{\alpha}_{2m+\mu'}\, Y^1{}_m(\theta_p, \phi_p) \qquad (7)$$

and $C_m^{\mu'}$ is given by

$$\sqrt{\frac{3}{10\pi}}\, C^{\mu'}{}_m = \begin{array}{c|ccc} m \backslash \mu' & -1 & 0 & 1 \\ \hline -1 & 1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{2}} & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{2}} \\ 1 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & -1 \end{array} \qquad (8)$$

The additional condition to be satisfied by the new velocity potential Φ is

$$-\frac{\partial\Phi}{\partial r'}\bigg|_{r'=\delta} = \text{velocity of the sphere in that direction}$$

$$= -\frac{\dot{a}-i\dot{b}}{2}\sqrt{\frac{8\pi}{3}}\, Y^1{}_1(\theta',\phi') + \dot{c}\sqrt{\frac{4\pi}{3}}\, Y^1{}_0(\theta',\phi') + \frac{\dot{a}+i\dot{b}}{2}\sqrt{\frac{8\pi}{3}}\, Y^1{}_{-1}(\theta',\phi') \quad \ldots (9)$$

So we take Φ in the form

$$\phi = -\left(r' + \frac{D}{r'^2}\right)\sum_{\mu'=-1}^{1} A_{\mu'}\, Y^1{}_{\mu'}(\theta',\phi') - \tfrac{1}{2}\left(r'^2 + \frac{E}{r'^3}\right)\sum_{\mu=-2}^{2}\dot{\alpha}_{2\mu} Y^2{}_\mu(\theta',\phi')$$

$$+ \frac{\delta^3}{r'^2}\sum^{1} B_{\mu'}\, Y^1{}_{\mu'}(\theta',\phi') \quad \ldots (10)$$

By (9) and (10) one easily obtains D, E and $B_{\mu'}$.

To calculate the thrust experienced by the sphere we use Bernoulli's equation

$$\frac{p}{\rho} = \frac{\partial\Phi}{\partial t} - \frac{1}{2}\, q^2 \quad \ldots (11)$$

where ρ is the density of the fluid, q, p are the velocity and the pressure at any point. In our case δ^3 is of the order $\frac{1}{A}$, where A is the mass number of the nucleus. So retaining terms of the order $\frac{1}{A}$ and neglecting all higher order terms, it is easily seen that the force experienced by the small sphere due to the pressure and velocity distribution given by (11) and (10) respectively, is derivable from a potential function and the dominant terms are given by

$$V = -\frac{3M\rho}{2\rho+4\sigma} r_p^2 \sum_{\mu=-2}^{2} \ddot{\alpha}_{2\mu} Y^2{}_\mu (\theta_p, \phi_p) \qquad \ldots \quad (12)$$

where M, σ are the mass and the density of the small sphere and the potential is measured from the surface of the nucleus. It is easily seen that the hamiltonian of the core will not change (up to the order of approximation we are concerned with) from that of A. Bohr (1952) who calculated the same with a velocity potential which is the same as our unperturbed Φ given by (5). Therefore up to $\frac{1}{A}$ order accuracy we may say that the potential energy of the particle at (r, θ, ϕ) within the nucleus, referred to its value at the surface, due to the surface oscillation is given by

$$H_{\text{Int}} = -K_1 . \frac{1}{2} Mr^2\omega^2 \sum_{\mu=-2}^{2} \alpha_{2\mu} Y^2{}_\mu \ (\theta, \phi) \qquad \ldots \quad (13)$$

where $K_1 = \frac{3\rho}{\rho+2\sigma}$ and $\omega = \omega_2$ of A. Bohr (1952). This is to be compared with the interaction hamiltonian (1) of A. Bohr. The matrix element of K in (1) has been estimated (A. Bohr (1953)) to be of the order of 40 Mev, whereas the corresponding term in case of our interaction comes out to be of the order of 2 Mev (taking $\sigma \simeq \rho$; M = mass of a proton). So it turns out that the classical value of the interaction strength is too small compared to what is required emperically. The empirical way of defining the core-to-particle interaction only at the surface (if it serves some purpose) is then perhaps justified, so far as the present model is concerned.

2. ANGULAR MOMENTUM

We calculate the angular momentum using the formula (A. Bohr (1952)).

$$\boldsymbol{m} = \int \rho(\boldsymbol{r} \times \boldsymbol{v}) d\tau \qquad \ldots \quad (14)$$

with

$$\boldsymbol{v} = -\operatorname{grad} \Phi \qquad \ldots \quad (15)$$

We write this as

$$\boldsymbol{m} = i B \sum_{\mu', \mu=-2}^{2} \dot{\alpha}_{2\mu'} \alpha^*{}_{2\mu} \boldsymbol{M}_{\mu', \mu} + \text{correction terms} \quad \ldots \quad (16)$$

where the first term in (16) is the same as that obtained by A. Bohr (1952) and the other leading terms are of the form

$$i\rho D\sum_{\mu'=-1}^{1}\sum_{s=-1}^{1}\sum_{\mu=-2}^{2}\sum_{t=1}^{3}\sum_{L=t-1}^{t+1}C_s^{\mu'}\dot{\alpha}_{2s+\mu'}\alpha^*_{2\mu}$$

$$\times\left\{\sum_{m=-t}^{t}\frac{3(2t+1)}{\sqrt{5(2L+1)}}\begin{pmatrix}t&1&2\\0&0&0\end{pmatrix}\begin{pmatrix}t&1&L\\0&0&0\end{pmatrix}\begin{pmatrix}t&1&2\\m&\mu'&m+\mu'\end{pmatrix}\begin{pmatrix}t&1&L\\m&s&s+m\end{pmatrix}r_p^2f_k(r_p,\delta)\right.$$

$$Y^L_{s+m}(\theta_p,\phi_p)M_{m+\mu',\mu}$$

$$-\sum_{m=-2}^{2}\sqrt{\frac{45}{2L+1}}\begin{pmatrix}2&1&t\\0&0&0\end{pmatrix}\begin{pmatrix}t&1&L\\0&0&0\end{pmatrix}\begin{pmatrix}2&1&t\\m&\mu'&m+\mu'\end{pmatrix}\begin{pmatrix}t&1&L\\m+\mu'&s&m+\mu'+s\end{pmatrix}r_p^2f_k(r_p,\delta)$$

$$\left.Y^L_{m+\mu'+s}(\theta_p,\phi_p)M_{m,\mu}\right\}$$

where $\begin{pmatrix}j_1&j_2&j\\m_1&m_2&m\end{pmatrix}$ is the Clebsch-Gordan coefficient $\langle j_1\,j_2\,m_1\,m_2\mid j_1j_2jm\rangle$ of Condon and Shortley (1953), $f_k(r_p,\delta)$ is some function of r_p and δ, and $D=\frac{1}{2}\delta^3$.

Magnetic moment is given by, (in the notation of *E*.Feenberg 1955)

$$\langle\nu\,j\,k;\,II\mid m_z\mid\nu j\,k\,;I\,I\rangle$$

where j, k, I are the total angular momentum of the extra-core particle, core and the nucleus respectively, ν is the phonon number. For the nuclei with $j=I$, and in case of one phonon excitation the contribution from the correction term becomes

$$F\begin{pmatrix}I&2&I\\I-1&1&I-1\end{pmatrix}^2\sum_{t=1}^{3}\sum_{L=t-1}^{t+1}\begin{pmatrix}I&L&I\\I-1&0&I-1\end{pmatrix}\begin{pmatrix}l&L&l\\0&0&0\end{pmatrix}\begin{pmatrix}t&1&2\\0&0&0\end{pmatrix}\begin{pmatrix}t&1&L\\0&0&0\end{pmatrix}^2\begin{pmatrix}t&1&2\\0&1&1\end{pmatrix}$$

$$\times U(\tfrac{1}{2}l\,I\,L;\;I\,l)(2t+1)\quad f_k(r_p,\delta)$$

where l is the orbital angular momentum of the extra-core nucleon, F is a constant of the order $\frac{1}{A}$, U is the U-coefficient of Jahn (1952), and in this expression we have replaced the matrix element of r_p by its average value. It is seen from this expression that for $j=I=1/2$ the correction term does not contribute anything to the magnetic moment, and for all other I there is a non-zero contribution. But as the factor F is of the order $\frac{1}{A}$, it turns out that the contribution affects only the second place in decimals, and as such the results of A. Bohr (1952, 1953) remain practically unchanged. It can also be verified that the results of quadrupole moments of A. Bohr also remain practically unaltered, if our form of interaction is superimposed on the kind of surface interaction used by him. So it may be concluded by saying that the semi-rigid model of nucleus does not affect the results of A. Bohr, so far as the points which we have investigated here are concerned.

N.B. It should be noted that the semi-rigid model of nucleus that we have used here is a bit idealisation of the same proposed by Moszkowski (1956).

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